

# Analysis of a Dynamic Voluntary Contribution Mechanism Public Good Game

23rd IPE Conference

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Why Solving the game is important?

To know optimal balance between investing in productive capacity and contributing to provision.

Examples of public goods

- Good environment
- National Defense



# The Game

- 4 people in group for 10 periods
- Each period has two stages:
  - 1 investment stage
  - 2 contribution stage
- Endowments of 10 for each player in each period



# Investment Stage

- Players can increase their contribution productivity from the starting value of 0.30
- Vote (median rule) to determine the amount each player in the group will invest in increasing contribution productivity
- Contribution productivity increases by 0.01 multiplied by the investment

$$\text{Contribution productivity} = M_t = M_{t-1} + 0.01 \cdot I_t$$

$$\text{for } t = [1..10]$$

$$M_0 = 0.3$$



# Contribution Stage

Players decide how to allocate their remaining money between private consumption and public good.

Payoff:

$$\pi_{it} = \omega - I_t - c_{it} + M_t \sum c_{jt}$$



# Example

Example ( $M_0 = 0.3$ ):

Table

Players	$\omega$	$I_t$	$M_t$	$C_{it}$	$M_t \sum c_{jt}$	$\pi_{it}$
1	10	3	0.33	7	4.95	4.95
2	10			5		6.95
3	10			3		8.95
4	10			0		11.95



# Potential Outcomes

- **The Lowest Payoff outcome.** How would the players act to get the lowest possible payoffs? What are the lowest possible payoffs?
- **The Nash Equilibrium.** What would happen if each player acted in his own interest?
- **The Socially Optimal outcome.** How should the players act so that the sum of payoffs is maximized? What is this sum of payoffs?





# Lowest payoff outcome

- Lowest possible payoff is 0
- Occurs if the group invests everything in every period and never contributes anything

Payoffs are 0 in every period and 0 at the end of 10 periods.

$$\pi_{it} = \omega - I_t - c_{it} + M_t \sum c_{jt}$$



# Nash equilibrium

- Think of the last period
- Player maximizes his payoff. If he contributes anything he reduces his payoff. Decides not to contribute.
- All players follow the same strategy
- If nobody contributes, then nobody invests



# Nash equilibrium

- Everyone is left with his endowment
- Occurs for all previous periods up to the first one
- All players follow the same strategy
- Nash equilibrium is for everyone to keep his money

Each person's payoff is  $10 \cdot 10 = 100$ .



# The mathematical model

$$f(I, C) = [\omega - C - I] + [4 \cdot M_t \cdot C]$$

$$M_t = M_{t-1} \cdot (1 + 0.01 \cdot I)$$

$$M_0 = 0.3$$

$4 \cdot M_t \cdot C$  is payoff and  $\omega - C - I$  is the amount left after both stages.



# Assumption

**Assumption:** the optimal result requires contributing all that is left after the investment.

We can eliminate one of the two variables -  $C$  or  $I$ .

Now  $I = p \cdot \omega$  and  $C = (1 - p) \cdot \omega$ .

$$f(p) = 4 \cdot M_t \cdot \omega \cdot (1 - p)$$

$$M_t = M_{t-1} \cdot (1 + 0.01 \cdot \omega \cdot p)$$

$$M_0 = 0.3$$

where:

- $p$  is the *proportion* of investment
- $\omega$  is the endowment (10)
- $M_t$  is the  $t_{th}$  multiplier



# Final model

From now, let us solve it specifically for our case, when endowment is 10.

$$f(p) = 40 \cdot M_t \cdot (1 - p)$$

$$M_t = M_{t-1} \cdot \left(1 + \frac{p}{10}\right)$$

$$M_0 = 0.3$$



# Approximation

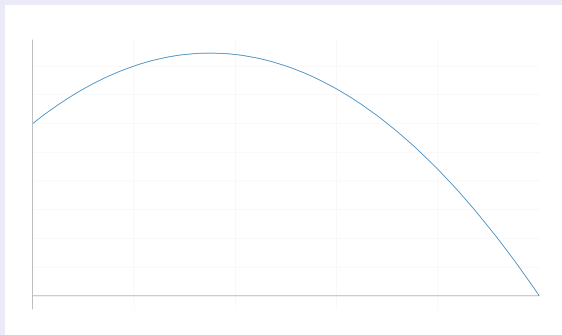
- Ran the simulation for 10 periods with step 0.1
- Time complexity of the algorithm would be  $O(n^a)$

**Assumption:** The optimal solution requires that players first only invest then only contribute.



# Graphical representation of computational result

Plotted by <https://plot.ly/plot>





# Regression analysis result

$$f(x) = 400 \cdot [-m \cdot \omega \cdot x^2 + (m \cdot \omega \cdot T - M_0) \cdot x + M_0 \cdot T]$$

$$x_{\max} = \frac{T}{2} - \frac{M_0}{2 \cdot m \cdot \omega}$$

$$f_{\max} = f(x_{\max}) = f\left(\frac{T}{2} - \frac{M_0}{2 \cdot m \cdot \omega}\right)$$

where:

- $m$  is the increase in contribution productivity (0.01)
- $T$  is the number of periods
- $x$  is the stage when players switch to contributing. The number before the decimal point defines a period. The number after the decimal point defines an investment in that period.



## In our specific case

$$f(x) = -0.1x^2 + 0.7x + 3$$

$$x_{\text{optimal}} = 3.5$$

which indicates investment until the 4<sup>th</sup> period and in that period investment of 5

$$f_{\text{optimal}} = f(x_{\text{optimal}}) = 169$$

which implies the payoff of 169.



# Thank you!

Questions?



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